INDUSTRIAL RESONATOR MUFFLER DESIGN

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ABSTRACT

This project covers an examination of the aspect ratio of a single resonator and the relationship between two or more resonators located close to each other. Two types of resonators will be examined; Helmholtz Resonator and Quarter Wave Tube. These resonators are actually side branch cavities connected to the main duct via an orifice. Basically, they function by placing very low impedance in parallel with the impedance of the remainder of the line at its point of insertion. The resonator impedance consists of the combination of capacitance, due to the cavity volume and inductance and resistance through the constricting orifice. It was found that two resonators in close proximity at similar resonant frequency will interact and can lead to a decrease in the overall performance compared to that of a single resonator. To assist with the result verification, the experimental results will be compared with Finite Element Analysis (FEA) in which FEA will model the duct under perfect condition.

KEYWORDS : Helmholtz Resonator, Quarter Wave Tube, Transmission Loss

1. INTRODUCTION

In the field of passive noise control in suppressing noise flowing through a duct, most of the formal contributions were related to single resonators and the relationship between two closely spaced resonators was not experimentally studied. Cazzolato, Howard & Hansen (2000) have presented Finite Element based relationship between two closely spaced resonators and the estimation of the resonant frequencies of rhomboidal shaped resonators. The approaches did not give a prediction of the performance for multiple resonators in an array. Besides, the aspect ratio and geometry of the throat of the Helmholtz did not verify substantially; however, the dimensional limits of the resonators are still valid.

Traveling wave through a duct consists of incident and reflected traveling components. Therefore, decomposition of the broadband stationary random signal into its incident and reflected components would be essential using the transfer function method, which was presented by J. Y. Chung and D.A. Blaser (1980) and ASTM Standards, E1050 (1998). The ASTM Standard is only applied for an impedance tube in which transmission loss(TL) was not presented in this paper. However, some particular standards such as spacing between microphones can be applied in this project to determine the transfer function between the microphones at upstream and downstream of the duct. The TL due to a resonator in the duct can be determined by using the complex reflection coefficient at upstream and downstream of the duct provided by J. Y. Chung and D.A. Blaser (1980).

2. BACKGROUND

As mentioned earlier, a stationary random acoustic plane wave can be decomposed mathematically into its incident and reflected components[4]. Transfer function relation between the acoustic pressures at two microphones will be needed and this can lead to the determination of the TL of the duct.

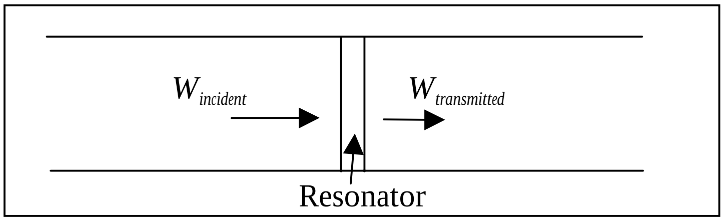

Fig. 1: Sound power incident, W_i and reflected, W_t through a resonator

Let R_u and R_d be the upstream and downstream complex reflection coefficients, therefore,

sound power incident, $W_i = S_{uu} \bullet A_u / (\rho c|1+R_u|^2)$ (1)

sound power transmitted, $W_t = S_{dd} \bullet A_d / (\rho c|1+R_d|^2)$ (2)

where:

S_{uu} = Auto-spectrum at the upstream measurement
A_u = Cross-sectional area at the upstream measurement
S_{dd} = Auto-spectrum at the downstream measurement
A_d = Cross-sectional area at the downstream measurement

Transmission loss is defined as the difference in sound power incident on and that transmitted past the resonators. Therefore, the transmission loss can be defined as follows:

$$TL = 10 \log_{10} (W_i / W_t) \quad (3)$$

From (1), (2) and (3),

$$TL = 20\log_{10}\left|\frac{H_r - H_{12}^u}{H_r - H_{12}^d}\right| - 20\log_{10}|H_t| + 10\log\frac{A_u}{A_d} \quad (4)$$

where:

H_{12}^u and H_{12}^d are the transfer function measure at the upstream and downstream locations

$|H_t| = \left|\frac{S_{dd}}{S_{uu}}\right|^{\frac{1}{2}}$ is the transfer function of the acoustic element within the range of linear acoustic propagation.

Basically, the geometry of the resonator is based on the values provided by Cazzolato, Howard & Hansen (1996).

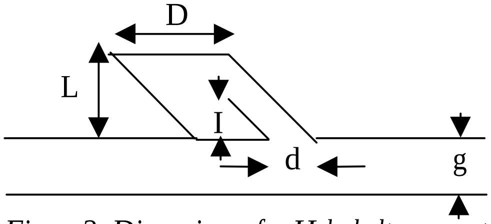

Figure 2: Dimensions of a Helmholtz resonator

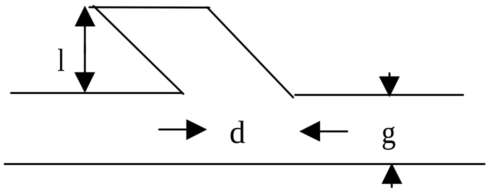

Figure 3: Dimensions of a quarter wave tube resonator

2.1. Helmholtz resonator

Resonance frequency of a resonator can be calculated using the equation provided by Panton & Miller (1975). Effective throat length must be obtained first in order to get the resonance frequency. The effective throat length can be calculated from the expressions below:

$$l_{eff} = 0.0835 + 0.1358D + 0.4334L - 0.0502\ln(L) + 0.0140\ln(g) + 0.0493\ln(d) + l\,[3.8871 - 9.6155d + 0.7403\ln(D) + 0.3997\ln(L) - 9.6110LD] \quad (5)$$

After obtaining the effective throat length, l_{eff}, resonance frequency can be calculated by using the expression below:

$$fr = \frac{c}{2\pi}\sqrt{\frac{S}{Vl_{eff} + \frac{1}{3}L^2 S}} \qquad (6)$$

which is applicable to resonators which satisfy $\kappa L < \pi/2$; $L < \lambda/4$ and where κ is the wavenumber, l_{eff} is the effective throat length, V is the volume of the cavity, S is the throat area and L is the depth.

2.2 Quarter wave tube

Same theory can be applied to Quarter wave tube by using the different equation provided[3]:

$$l_{eff} = -0.059 + 1.5845l - 0.1432l^2 + 0.2028d + \frac{0.205g}{l} - \frac{0.0040l}{g} \qquad (7)$$

where all dimensions are in meters and $c = 407\text{m/s}$.

After obtaining the effective throat length, l_{eff}, resonance frequency can be calculated by using the equation provided by Panton & Miller (1975):

$$f_r = \frac{c}{4l_{eff}} \qquad (8)$$

The calculated resonance frequency will then be compared with resonance frequency found from the experiment.

3. EXPERIMENTAL STUDY

The geometry of the duct for this project is shown as below:

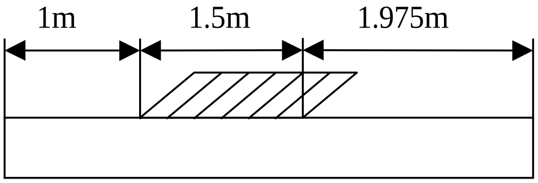

Fig. 4: Length of resonators

The experimental model is built of 25mm thick Medium Density Fibreboard (MDF), with a length of 4.5meters. The maximum inner area of the duct is 0.375m x 0.5m while the minimum area of the duct can be adjusted to 0.05m x 0.5m. The interested frequency range used is 100Hz to 1060Hz. A wedge, fabricated using Bradford Rockwool is used as the anechoic termination at the end of the duct. Random noise is used as the sound source.

TL can be calculated by using transfer function method which was discussed in the previous section. The TL found would then be compared with Finite Element Analysis (FEA) in which FEA will model the duct under perfect condition.

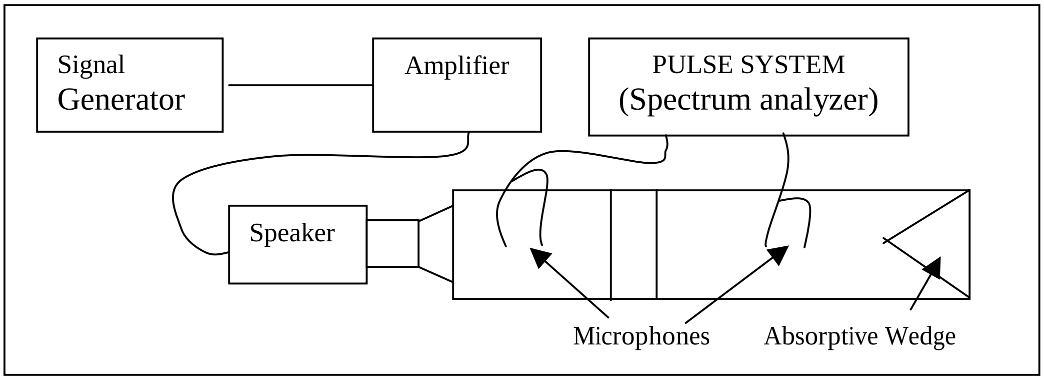

Fig. 5. Schematic Diagram of experimental set-up for Transmission Loss

4. RESULTS AND DISCUSSION

4.1. Experimental Result (Quarter Wave Tube)

For d = 0.093m, l = 0.12

Resonance Frequency = 392Hz
TL = 8.97dB

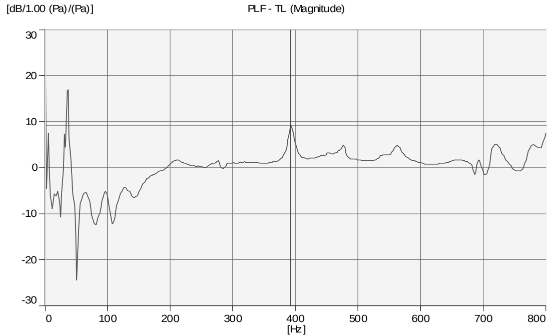

Fig. 6: Single resonator

Resonance Frequency = 406Hz
TL = 17.9dB

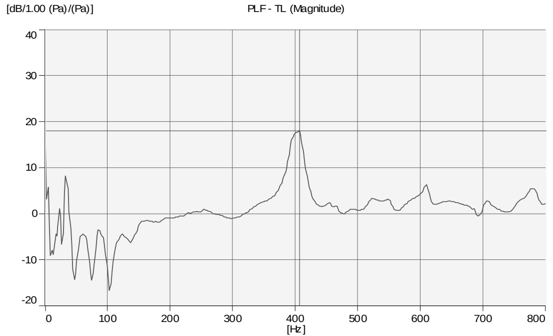

Fig. 7: Two resonators at 0.4m apart

Resonance Frequency = 400Hz
TL = 32.4dB

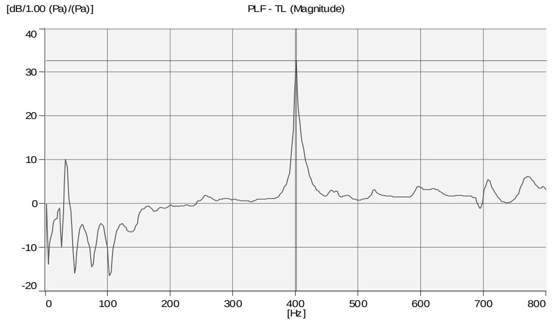

Fig. 8: Two resonators at 0.25m apart

Resonance Frequency = 400Hz
TL = 6.52dB

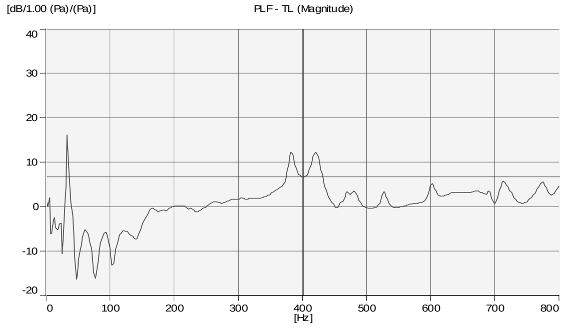

Fig.9 Two resonators at 0.15m apart

Fig.6 indicates that the TL for a single resonator is 8.97dB at a resonance frequency of 392Hz. Fig.7,8 and 9 were produced by using a pair of resonators of the similar dimensions but at different center distances apart. Fig. 7 and 8 shows that the TL increased when the two resonators are 0.4m and 0.25m apart. This is because when the resonator center distances are greater than $\lambda/4$ apart, the TL in dB is cumulative. However, when the two resonators are in close proximity, there is a significant decrease in TL as seen in Fig. 9. The TL is decreased because the resonators tend to cancel one another's effectiveness by a dipole effect when the center distance of the resonators is less than $\lambda/4$ apart[3]. In addition, the acoustic coupling between two closely spaced resonators produces a shift in the resonance frequencies, producing two distinct frequencies; one lower and one higher than the original frequency as seen in Fig. 9. This "de-tuning " of the resonators may have a detrimental effect on the overall performance of the silencer by creating "holes" in the TL spectrum and is particularly apparent when the resonators have similar resonant frequencies[3].

4.2. Comparison between resonance frequency

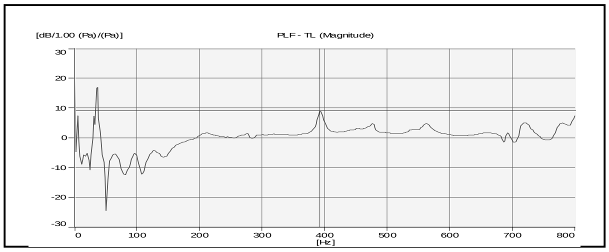

Fig. 10: Experimental result

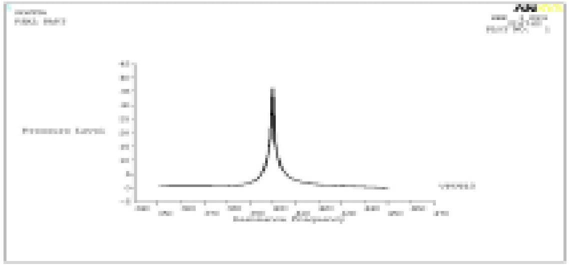

Fig. 11: Resonance frequency by using FEA

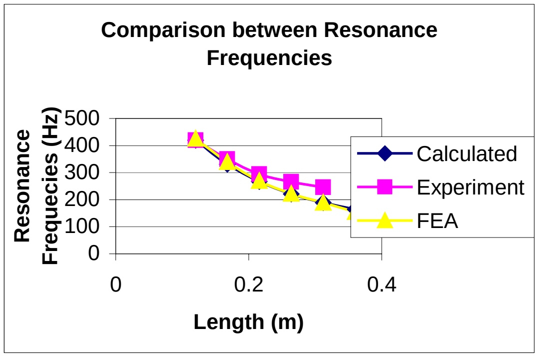

Fig. 12: Comparison between resonance frequency (d = 0.043m)

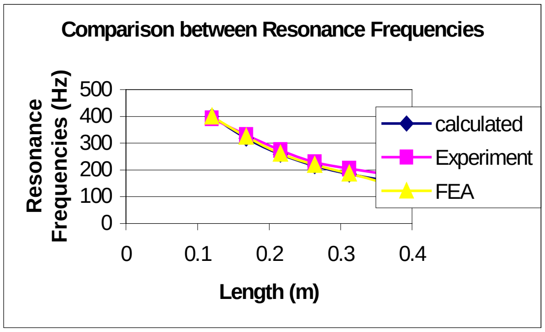

Fig. 13: Comparison between resonance frequency (d = 0.093m)

a) width of the resonator, d = 0.043m

	Resonance Frequency		
Aspect Ratio	**Calculated**	**Experiment**	**FEA**
0.3583	419.8320946	420	426
0.2986	372.5253594		
0.2560	330.9764643	350	339
0.2240	295.9289748		
0.1991	266.6474243	294	270
0.1792	242.1276193		
0.1629	221.4525604	266	222
0.1493	203.8695256		
0.1378	188.7828808	246	189.5
0.1280	175.7264804		
0.1194	164.335515	N/A	154.5
0.1120	154.3230149		
	Resonance		

Table 1: Resonance frequency

b) width of the resonator, d = 0.093m

	Frequency		
Aspect Ratio	**Calculated**	**Experiment**	**FEA**
0.7833	399.59715	392	400
0.6528	356.506652		
0.5595	318.270785	330	324
0.4896	285.730208		
0.4352	258.338765	274	260.5
0.3917	235.257077		
0.3561	215.691302	228	219
0.3264	198.976713		
0.3013	184.57996	204	186.5
0.2798	172.079196		
0.2611	161.141458	186	152.5
0.2448	151.502971		

Table 2: Resonance frequency

By using equation (8), theoretical resonance frequency for different width of the resonator can be determined. By judging the trend of the data compiled above, it can be concluded that the resonance frequency is inversely proportional to the increasing length of resonators. From the experimental result shown above, the values do not follow the trend of the calculated values. This might be due to some errors, which were introduced during conducting the experiment. Anyway, the actual reason for the differences is still being investigated. The results found from FEA is quite similar to the calculated values as it was conducted under ideal conditions.

The resonance frequencies obtained from FEA and experimental are different from one another and from the theoretical results calculated. This could be due to the fact that FEA modeling is assumed as an ideal case as no external factors affects the system. On the other hand, the differences in the experimental results could be affected by a number of factors. One factor, which might have caused the differences in the data collected, is due to the leakage of sound through gaps. Another factor is due to the coupling of the sound with the panels. However, both FEA and experimental results show that as the length of the resonator increases, the transmission loss obtained across the resonators decreases as well as the resonance frequencies. Both the results also show that as the width, d, increases, the transmission loss increases but the resonance frequency decreases.

5. CONCLUSION

When identical quarter wave tube resonators are positioned less than $\lambda/4$ apart, interactions between these resonators reduced the transmission loss significantly compared to that obtained when the same resonators are separated by more than $\lambda/4$ apart. However, FEA modeling was unable to verify the experimental results of interactions between identical resonators as the results obtained from FEA was not satisfactory. Due to time constraints, the errors in FEA modeling of the identical resonators were unable to be corrected. In addition, studies on Helmholtz resonators are not covered in this seminar paper as there was insufficient time to conduct the tests. However, more tests will be carried out to verify the results obtained in the above experiment.

6. REFERENCES

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